List of Topics for the CAM Preliminary Exam

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Numerical linear algebra

Linear algebra review

1. The field of complex numbers
2. Vector spaces
3. Normed spaces
4. Inner product spaces
5. Gram–Schmidt orthogonalization
6. Linear operators and matrices
7. Matrix norms
8. Eigenvalues and spectral decomposition

The singular value decomposition

1. Reduced and full singular value decompositions
2. Existence and uniqueness of the SVD
3. Further properties of the SVD
4. Low rank approximations

Systems of linear equations

1. Solution of simple systems
2. LU factorization
3. Gaussian elimination with column pivoting
4. Implementation
5. Special matrices

Norms and matrix conditioning

1. The spectral radius
2. Conditioning

Linear least squares problem

1. Linear least squares: Full rank setting
2. Projection matrices
3. Linear least squares: The rank–deficient case
4. QR Factorization
5. The Moore–Penrose pseudo–inverse
6. The modified Gram–Schmidt process
7. Householder reflectors

Linear iterative methods

1. Linear iterative schemes
2. Spectral convergence theory
3. Matrix splitting methods
4. Richardson's method
5. Relaxation methods
6. The Householder–John criterion
7. Convergence in energy norm

Variational and Krylov subspace methods

1. Basic facts about HPD matrices
2. Gradient descent methods
3. The steepest descent method
4. The conjugate gradient method

Eigenvalue problems

1. Estimating eigenvalues using Gershgorin discs
2. Stability
3. The Rayleigh quotient for Hermitian matrices
4. Power iteration methods
5. Reduction to Hessenberg form
6. The QR method

Nonlinear equations and optimization

Solution of nonlinear equations

1. Bisection method
2. Fixed points and contraction mappings
3. Newton's method in one space dimension
4. Newton's method in several dimensions

Initial value problems for ordinary differential equations

Single–step methods

1. Single–step approximation schemes
2. Consistency and convergence of some single–step approximation

Runge–Kutta methods

1. Simple two–stage schemes
2. Definition and basic properties
3. Collocation methods

Linear multi–step methods

1. Consistency
2. Adams–Bashforth and Adams–Moulton methods
3. Backward differentiation formula methods
4. Zero stability
5. Convergence of linear multistep methods
6. Dahlquist theorems

Stiff systems of ordinary differential equations and linear stability

1. The linear stability domain and A–stablity
2. A–Stability of Runge–Kutta schemes
3. A–stability of linear multi–step methods

Boundary and initial boundary value problems

Finite difference methods for elliptic problems

1. Grid functions and finite difference operators
2. Consistency and stability of finite difference schemes
3. The Poisson problem in one dimension
4. Elliptic problems in one dimension
5. The Poisson problem in two dimensions

Finite element methods for elliptic problems

1. The Galerkin method
2. The finite element method in one dimension
3. The finite element method in two dimensions

Approximation of the diffusion equation

1. Diffusion in 1D
2. $L^\infty_\tau(L^\infty_h)$ stability and convergence
3. $L^\infty_\tau(L^2_h)$ stability and convergence
4. An advection-diffusion equation
5. An energy method
6. von Neumann stability analysis

The advection and wave equations

1. The linear advection equation
2. Positivity and max-norm dissipativity
3. Advection on a periodic spatial domain
4. The wave equation and hyperbolic systems

References

Main

- A.J. Salgado, S.M. Wise. *Graduate Numerical Analysis: A Modern Introduction*. Set of notes available at `http://www.math.utk.edu/~swise/Site/Numerical_Analysis.html`.

Supplementary

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