Math Ecology Preliminary Exam Topics

Deterministic Models: single biological population and community model formulation, interpretation, stability (various tests), types of bifurcations, units, nondimensionalization, types of growth functions and functional responses, density dependence, showing positivity of compartments when appropriate

1. Single differential equation model, including bifurcation analysis, harvest models with maximal sustainable yield

2. Single discrete model, including periodicity, cycles, order of events and system of discrete equations

3. discrete, integrodifferential equations, and ODE models with delays, including oscillation behavior

4. Model systems of ODEs, illustrating predator-prey (also effect of harvesting), cycling, chemostat models, competition models, mutualism, Dulac's criterion, phase plane diagrams

5. Optimal control problems for models with one differential equation with one control, including formulation, interpretation, and necessary conditions with payoff terms and bounded controls

6. Spatial models with PDEs:

   understanding of diffusion, advection and growth terms and types of boundary conditions

   FINITE SPATIAL DOMAIN: linear PDEs and solutions by separation of variables, critical patch size (for KISS, radial KISS and parabolic decline), positive steady states for Fisher PDEs, phase-plane approaches for nonlinear problems

   INFINITE SPATIAL DOMAIN: fundamental solution of heat equation, speed of propagation, speed of spread from a point source (using detectable level), radial symmetric case, traveling wave solutions for Fisher PDE, minimum wave speed, applications to infectious disease models

7. Age structure models

   Euler Lotka equation, characteristic equation, age-specific mortality and natality, characteristic equation, dominant eigenvalue, stable age distribution, net reproductive rate, geometric growth rate, average age of reproduction, Perron-Frobenius Theorem, digraphs of discrete, linear, stage-structured models

   Four types of models: Lotka integral equation, difference equation, Leslie Matrix, McKendrick-von Foerster PDE (method of characteristics), formulation and mathematical connections between these models.

Stochastic Models: single biological population formulation, interpretation, long term properties

1. birth and death processes, including probability generating function (also deriving and solving PDE for this function), extinction time, expected value, variance

2. branching processes with probability generating function, extinction probability, and population growth rate.

Be able to answer essay questions relating to features in models and give explanations of representations of biological mechanisms. Features may include spatial, stochastic, movement, density dependence, interaction terms, and functional responses. Be able to state examples of models to illustrate your points.

Most of these topics are from Kot's book, Elements of Mathematical Ecology, chapters 1 - 24 and age-independent models section in chapter 25.

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