THE 52nd JOHN H. BARRETT MEMORIAL LECTURES

SCHEDULE

The lectures, discussion activities, breakfast and coffee breaks will take place in the Science and Engineering Research Facility SERF 307

Lunch and dinner will be provided in Ayres 401.

Monday, May 20

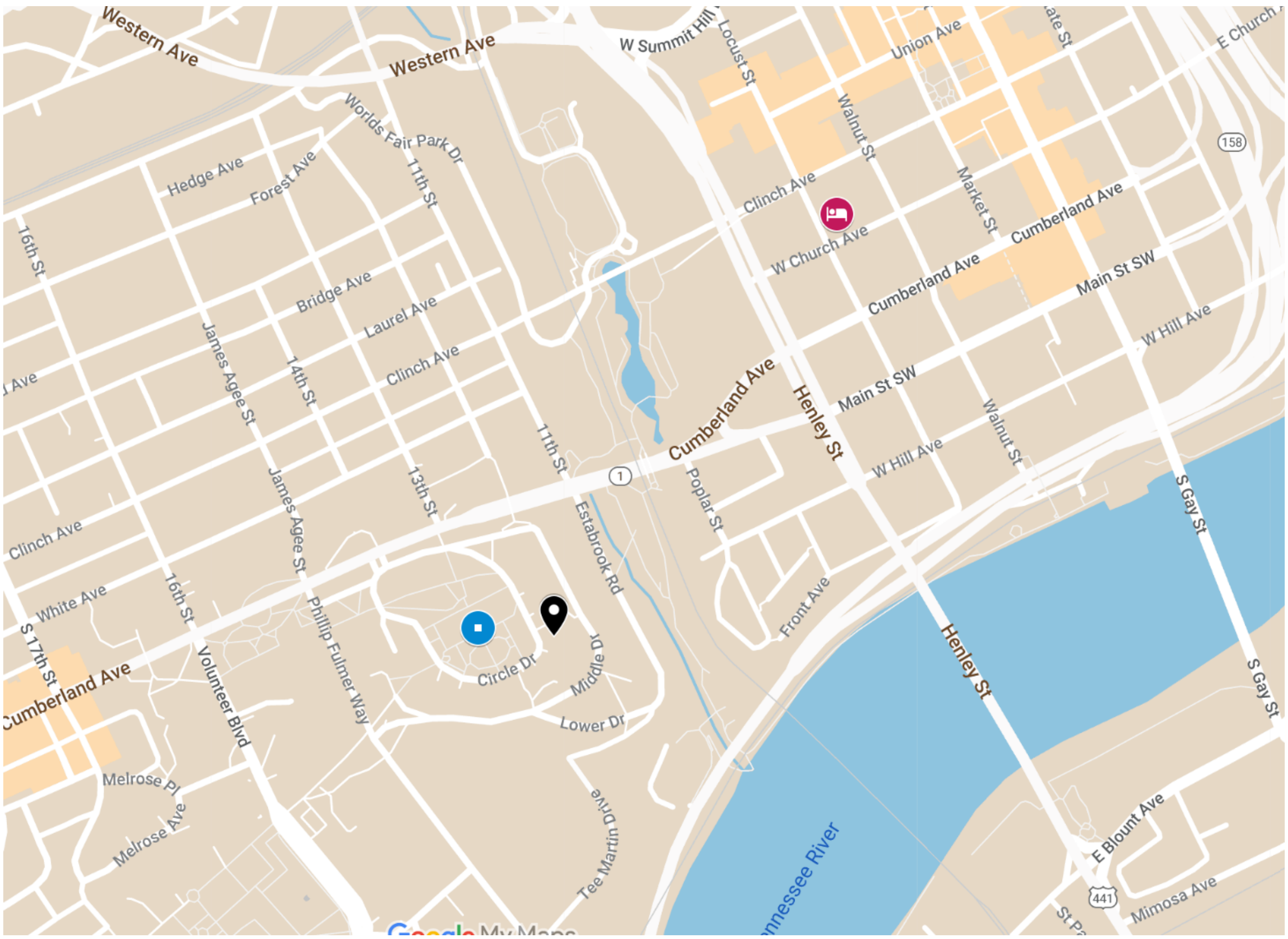
8:30 – 9:20	Check in/Light breakfast
9:20 – 9:30	Opening remarks by the Department Head
9:30 – 10:30	**Jeremy Quastel**
	Integrable fluctuations in random growth
10:30 – 11:00	Coffee break
11:00 – 12:00	**Yu Gu**
	Integration by parts in KPZ
12:00 – 1:30	Lunch at Ayres 401
1:30 – 2:30	**Tai Melcher**
	Things I've learned from the Heisenberg group
2:30 – 3:00	Coffee break
3:00 – 4:00	**Sandra Cerrai**
	On the Smoluchowski-Kramers approximation for systems of stochastic wave equations with state-dependent friction

Tuesday, May 21

9:00 – 9:30	Light breakfast
9:30 – 10:30	**Christian Houdré**
	Limiting laws in some subsequences problems
10:30 – 11:00	Coffee break
11:00 – 12:00	**Yimin Xiao**
	Collision of eigenvalues of random matrices with Gaussian random field entries
12:00 – 1:30	Lunch at Ayres 401
1:30 – 2:30	**Carl Mueller**
	Valleys for the stochastic heat equation
2:30 – 3:00	Coffee break
3:00 – 4:00	**Hakima Bessaih**
	Various numerical scheme for stochastic hydrodynamic models
4:00 – 5:30	Free time
5:30 – 7:30	Dinner at Ayres 401

Wednesday, May 22

9:00 – 9:30	Light breakfast
9:30 – 10:30	**Xue-Mei Li**
	Long range dependent noise
10:30 – 11:00	Coffee break
11:00 – 12:00	**Jonathan Mattingly**
	Two vignettes: Gaussian structure of stochastic Burgers and optimal mixing on the torus
12:00 – 12:15	Ending



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Science and Engineering Research Facility, 1414 Circle Dr, Knoxville, TN 37916

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Hakima Bessaih, Florida International University

Title: *Various numerical scheme for stochastic hydrodynamic models*

Abstract: We will consider various models in hydrodynamic, including the 2d Navier-Stokes, Boussinesq equations, and a Brinkman-Forchheimer-Navier-Stokes equations in 3d. These models are driven by an external stochastic Brownian perturbation.

We will implement space-time numerical schemes and prove their convergence. We will show some rates of convergence as well. Furthermore, we will show the difference between the stochastic and the deterministic cases.

Sandra Cerrai, University of Maryland

Title: *On the Smoluchowski-Kramers approximation for systems of stochastic wave equations with state-dependent friction*

Abstract: I will study the small-mass limit, known as the Smoluchowski-Kramers approximation, for systems of stochastic damped wave equations in the case the friction is non-constant. I will describe what was known in the case of a single equation and how, dealing with systems makes the analysis more delicate and requires new ideas and techniques.

Yu Gu, University of Maryland

Title: *Integration by parts in KPZ*

Abstract: The KPZ equation serves as a default model for surface growth subject to random perturbations. It's well-known from the seminal work of Bertini-Giacomin in 97 that an invariant measure of the 1d KPZ equation is the two-sided Brownian motion. Their proof relied on a discrete approximation through the asymmetric simple exclusion process. In this talk, I will present a more analytic proof, based on Stein's method and integration by parts, and I will also explain why we would prefer this approach. This is based on joint work with Jeremy Quastel.

Christian Houdré, Georgia Tech University

Title: *Limiting laws in some subsequences problems*

Abstract: I will survey various results on the limiting law of the, properly centered and normalized, length of the longest common/ and or increasing subsequences in random words. These limiting laws range from the Gaussian one to the maximal eigenvalues of some Gaussian random matrices, to less explicit ones. The survey will end with some open questions.

Xue-Mei Li, EPFL and Imperial College London

Title: *Long range dependent noise*

Abstract: In stochastic modeling, the traditional reliance on uncorrelated Gaussian noise has been a staple in both stochastic differential equations and stochastic partial differential equations. Recently, however, long-range dependent properties have captured significant attention, particularly in statistical analysis, and are now regularly incorporated into stochastic analysis. Notable examples of noise with long-range time dependence, such as fractional Brownian motions, Hermite processes, and Volterra processes, are increasingly integrated into multi-scale stochastic models. Rapid advancements in Rough Path Analysis have enabled comprehensive approaches to issues related to ergodicity and effective dynamics. Additionally, long-range spatial dependent noise has gained prominence, especially evident in applications like stochastic heat equations where the fluctuation problem associated with diffusively scaled solutions from their average exhibits distinct characteristics compared to those from compactly supported correlation scenarios. Unlike compactly supported correlations – which are typically known to dissipate at large scales – long-range dependence is maintained in the large-scale scaling limit. This talk will explore these recent developments and discuss their profound implications in this rapidly evolving field

Jonathan Mattingly, Duke University

Title: *Two vignettes: Gaussian structure of stochastic Burgers and optimal mixing on the torus*

Abstract: If time permits I will give two vignettes that use stochastic analysis.

In the first I will show that the law of the stochastic burgers equation at a fixed time t is absolutely continuous with respect to the natural Gaussian measure on the spatial domain. The results will apply to forcing just up to the point where the roughness of the forcing corresponds to the classical KPZ equation in the Burgers setting.

If time permits, I will then give proof that transporting by a velocity fields consisting of alternating shears coupled with diffusion provides the optimal mixing rate at as the diffusion coefficient is sent to zero. Though the result is deterministic, the proof uses a representation of the flow as a random dynamical system and leverages a mixture of Classical Dynamical systems, Stochastic Dynamics, and PDE methods that mix a Lagrangian and an Eulerian perspective.

Tai Melcher, University of Virginia

Title: *Things I've learned from the Heisenberg group*

Abstract: A collection of vector fields on a manifold M is said to satisfy the Hormander condition if they generate the span of the tangent space at all points of M. Such a collection has a naturally associated diffusion with generator the sum of squares of the vector fields, and often induces a natural geometry on M via the horizontal distance. In finite dimensions, the associated diffusions are known to enjoy various regularity properties, and it's known that the topology induced by the horizontal distance is equivalent to the intrinsic (locally Euclidean) one. The situation in infinite dimensions is more complicated and less understood.

Finite-dimensional Heisenberg groups are model spaces for studying properties of distances and diffusions associated to Hormander collections of vector fields. We'll discuss some things we've learned from studying infinite-dimensional Heisenberg groups. This includes an analytic condition which is equivalent to the Hormander condition in this setting. Under this condition, we may prove regularity properties of the associated diffusion but can show that the horizontal topology may no longer be equivalent. This construction also allows us to discuss the distribution of the central element of the diffusion, which is an infinite-dimensional stochastic Levy area.

Carl Mueller, University of Rochester

Title: *Valleys for the stochastic heat equation*

Abstract: This is joint work with Davar Khoshnevisan and Kunwoo Kim.

There has been an enormous amount of work on intermittency for the parabolic Anderson model:

$$\partial_t v(t,x) = \frac{1}{2}\partial_x^2 v(t,x) + v(t,x)\xi(t,x)$$

where $t > 0, x \in \mathbb{R}$, $v(0,x) = 1$, and $\xi(t,x)$ is two-parameter white noise. The solution v is dominated by tall peaks that become more and more widely spaced as $t \to \infty$. These peaks are well-studied, but we will focus on the valleys between the peaks.

We consider a more general equation, but in terms of the parabolic Anderson model above, our main result is that there exist positive constants Λ_1, Λ_2 such that for all t larger than some finite random time T, we have

$$\sup_{|x| \le \exp(\Lambda_1 t^{1/3})} v(t,x) \le \exp\left(-\Lambda_2 t^{1/3}\right)$$

almost surely. In other words, on a rapidly growing spatial interval, u rapidly decreases to 0.

Jeremy Quastel, University of Toronto

Title: *Integrable fluctuations in random growth*

Abstract: The last 25 years has seen considerable progress in understanding fluctuations of random growing interfaces. They are often described by the KPZ equation. But this is just one member of a huge universality class, and special models turn out to have a high degree of solvability. We will describe one such model, the polynuclear growth model, its solution, connections to integrable systems, and the long-time, large-space limit to the KPZ fixed point and its integrable structure.

Yimin Xiao, Michigan State University

Title: *Collision of eigenvalues of random matrices with Gaussian random field entries*

Abstract: Let $\xi = \{\xi(t) : t \in \mathbb{R}_+^N\}$ be a centered Gaussian random field and let $\{\xi_{i,j}, \eta_{i,j} : i,j \in \mathbb{N}\}$ be a family of independent copies of ξ. For $\beta \in \{1,2\}$ and a fixed integer $d \ge 2$, consider the following $d \times d$ matrix-valued process $X^\beta = \{X^\beta_{i,j}(t); t \in \mathbb{R}_+^N, 1 \le i,j \le d\}$ with entries given by

$$X^\beta_{i,j}(t) = \begin{cases} \xi_{i,j}(t) + \iota \mathbb{1}_{[\beta=2]}\eta_{i,j}(t), & i < j; \\ \sqrt{2}\xi_{i,i}(t), & i = j; \\ \xi_{j,i}(t) - \iota \mathbb{1}_{[\beta=2]}\eta_{j,i}(t), & i > j, \end{cases}$$

where $\iota := \sqrt{-1}$ is the imaginary unit. Thus, for every $t \in \mathbb{R}_+^N$, $X^\beta(t)$ is a real symmetric matrix for $\beta = 1$ and a complex Hermitian matrix for $\beta = 2$.

Jaramillo and Nualart (2020) provided a necessary condition and a sufficient condition for the collision of eigenvalues of X^β. Song et al (2021) extended their results to the case where k eigenvalues collide with $2 \le k \le d$ and determined the Hausdorff dimension of the set of collision times. However, in the case of "critical dimension", the problem whether the eigenvalues of X^β collide or not was left open.

We solve this problem by extending Talagrand's covering argument. More specifically, let $X = \{X(t), t \in \mathbb{R}^N\}$ be a centered Gaussian random field with values in $\mathbb{R}^d$ satisfying certain conditions

and let $F \subset \mathbb{R}^d$ be a Borel set. We provide a sufficient condition for F to be polar for X, i.e. $\mathbb{P}\big(X(t) \in F \text{ for some } t \in \mathbb{R}^N \backslash \{0\}\big) = 0$. Our new condition is related to the upper Minkowski dimension of F and is applicable to a variety of examples of Gaussian random fields including random matrices with Gaussian random field entries.

This talk is mainly based on a joint paper with Cheuk-Yin Lee, Jian Song, and Wangjun Yuan.